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Trajectory Planning and Control of a Limit Cycle Walker Based on a Receding Horizon Control Scheme

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Abstract

Investigation of the aspects of Limit Cycle Walking (LCW) is still a relatively unexplored field of study in comparison whit ZMP based walking robots. In this paper, for a limit cycle walker, a receding horizon based controller is implemented. The biped robot model is considered an under-actuated hybrid planar system with five degrees of freedom having two actuators. The model is hybrid because of the impacts of the foot with ground. So, there is a discontinuity (jumps) in the states of the robot and controller should tackle this problem. The main idea of control design consists in the choice of particular trajectories for the directly controlled degree of freedoms (using receding horizon scheme) for which the dynamics of indirectly controlled DOF (un-actuated coordinate) of the system tracks a desired trajectory. The controller is implemented to control the whole system to sustain its stable cyclic walking. This approach avoids the need to use Poincare-like argumentation in the proof of stability of limit cycles. Simulation results demonstrate the effectiveness of the method.

1. Introduction

Building of a realistic biped robot which can walk around its environment in a stable, efficient, and naturalistic manner has long been a goal of roboticists[1]. Generally the motion planning and control of biped robots is a very challenging problem because of; high degrees of freedom, high nonlinear dynamics, under-actuation and the hybrid nature of their dynamic resulting from impacts with the ground which produces discontinuities (jumps) in the states of the robot.

Previous studies on the modeling and control of bipeds can be classified into two categories:

1. Zero Moment Point (ZMP) based walking robots

2. Passive-dynamic walkers and limit-cycle walkers

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The first class of bipeds is controlled using the “Zero Moment Point” (ZMP) principle [2]. Today, this principle is a well-known concept and it is used in the gait control of most of the advanced bipedal robots, like Honda ASIMO [3] and HRP [4]. The motions which are achievable in this class of bipeds are highly conservative, inefficient, and unnatural looking.

The second broad class is a relatively new. Limit Cycle Walking is a nominally periodic sequence of steps that is stable as a whole but not locally stable at every instant in time. Inspired by the completely passive walkers of McGeer, 1990 [5], these robots forgo full actuation and allow gravity and natural dynamics to play a large part in the generation of motion. They may be completely passive, or partially actuated. Even with partial actuation, the motions generated can be life-like and highly efficient energetically [6]. McGeers pioneering work has inspired the creation of limit cycle walkers all over the world. The use of high feedback gains in ZMP based robots actively fights the natural dynamics of the system at the cost of extra energy expenditure. In contrast, Limit Cycle Walking allows the natural dynamics of a walking system to help ensure convergence to the desired motion. Limit cycle walkers do very well in terms of energy efficiency but still has limited disturbance rejection capabilities [7].

To summarize, while the potential of Limit Cycle Walking is remarkable, it is still a relatively unexplored field of study, and the operability of Limit Cycle Walking robots is not as developed as in robots based on the ZMP (or similar) concept.

The Human walking approach is based on optimal algorithms, which use some goals and constraints to displace the body from one point to another, while considering and predicting the environment changes, in order to decide adaptively to accomplish safe and without falling walk. A suitable way of imitating this behavior for motion control of the biped robot and online motion synthesis is the receding-horizon optimal control -also known Model Predictive Control (MPC) - techniques [8]. In recent years, researchers have applied this method for trajectory planning and control of biped robots [9]. Historically, this approach has been applied to large systems such as chemical plants whose dynamics were slow enough to be compatible with the required optimization time. The exponential growth of available computer power at constant cost has made it progressively possible to apply these methods to more rapid processes.

In this paper a receding horizon scheme is exploited for a three link limit cycle walker. A model for biped robot is considered which its characteristics are; planar, hybrid, nonlinear and under-actuated. The impact dynamic is considered in the modeling and a map is obtained to relate the states before and after impact. The contribution of this work is that, for above mentioned model a dynamic balance controller is implemented and investigated based on receding horizon scheme. The controller controls the whole system to sustain its stable cyclic walking. One of the advantages of the approach is that avoids the need to use Poincar´e-like argumentation in the proof of stability of limit cycles. Poincar´e-like method needs high cost offline computations.

2. Modeling

The modeling approach presented in this paper is closely related to the work ofWestervelt et al., in 2007 [10]. The three-link walker provides the simplest example where torso stabilization is important. The robot is considered bipedal and planar with five degrees of freedom. It is assumed to have two telescopic legs that are connected at hip by ideal revolute joints and are carrying the torso segment. There is a mass at the center of each leg and two masses at the hips and the end of a torso segment, respectively. Two torques, u1 and u2 are applied between the torso and the stance leg, and the torso and the swing leg, respectively. There is no torque at the contact point of the leg with the ground. The representative model structure is shown in Figure 1.

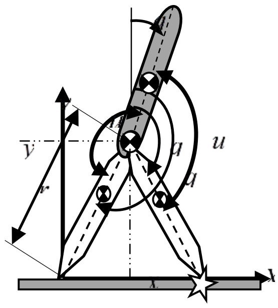

Figure 1. Schematic view of the three-link limit cycle walker

It is assumed that the walking cycle consists of successive phases of single support and double support taking place in an infinitesimal length of time. The model of the biped robot thus consists of two parts: the dynamics of the robot during the swing phase, and an impulse model of the contact event.

3. Dynamic equations of Swing Phase

During the swing phase of the motion, the stance leg is acting as a pivot, and thus there are only three degrees of freedom. Using Lagrange formulation [11], the mathematical model describing the biped moving in the sagittal plane is as follows

$$M(q)\ddot{q} + N(q,\dot{q}) + G(q) = Su + F_{ext} \tag{1}$$

where $M(q)$ is the inertia matrix, $N(q,\dot{q})$ contains the centrifugal and Coriolis forces terms, $G(q)$ is the vector of gravitational forces, $u = [u_1 \ u_2]^T$ is the vector of control inputs, S is a torque distribution matrix, q is the vector of generalized coordinates (Figure 1). F_{ext} is torques generated by external forces such as ground contacts. In single support phase it can be expressed as

$$F_{ext} = J_1^T(q)\lambda \tag{2}$$

Where $J_1^T(q)$ and λ represent the Jacobian matrix of the holonomic contact constraints, and the Lagrange multipliers of contact forces, respectively.

4. Dynamic Equations of Impact with Ground

The end of the swing phase is characterized by a collision between the swing foot and the ground which is modeled as a contact between two rigid bodies. There are many rigid impact models in the literature, and under reasonable hypotheses all of them can be used to obtain an expression for the velocity of the generalized coordinates after the impact of the swing leg with the walking surface in terms of the velocity and position just before the impact. The model from [12] is used here. The contact model requires the full five degrees of freedom of the robot. This gives once again a model of the form

$$M_e(q)\ddot{q} + N_e(q,\dot{q}) + G_e(q) = Su + \delta F_{ext} \tag{3}$$

We derive the impact equations under assumptions: 1) the impact takes place over an infinitesimally small period of time; 2) the external forces during the impact can be represented by impulses; 3) impulsive forces may result in an instantaneous change in the velocities of the generalized coordinates, but the positions remain continuous; and 4) the torques supplied by the actuators are not impulsional. With these assumptions, (3) is integrated over the “duration” of the impact to obtain

$$M_e(q)(\dot{q}^+ - \dot{q}^-) = F_{ext} = J_2^T(q)\lambda \tag{4}$$

where $F_{ext} = \int_{t^-}^{t^+} \delta F_{ext}(\tau)d\tau$ is the result of integrating the contact impulse over the impact duration, $\dot{q}^+$ is the velocity just after the impact and $\dot{q}^-$ is the velocity just before the impact. Since the positions do not change during the impact, $q^+ = q^-$. $J_2(q)$ is the Jacobian matrix of the cartesiancoordinates of the swing leg foot. Eq. (4) involves five constraints and seven unknowns F_{ext} and $\dot{q}^+$. Two additional equations may be obtained from the condition that the impacted leg does not rebound nor slips at impact, which is

$$y^+_{Swing} = \dot{y}^+_{Swing} = 0,\; x^+_{Swing} = \dot{x}^+_{Swing} = 0 \quad (5)$$

Above conditions can be expressed as

$$J_2^T(q)\dot{q}^+ = 0 \quad (6)$$

Eqs. (4) and (6) are linear in the unknowns and can be solved for $\dot{q}^+$, and λ. It is straightforward to verify that a unique solution always exists. The solution of (4)–(6) leads to

$$\begin{cases} \dot{q}^+ = [I - M^{-1}J_2^T(J_2M^{-1}J_2^T)^{-1}J_2]\dot{q}^- = D(q)\dot{q}^- \\ \lambda = [(J_2M^{-1}J_2^T)^{-1}J_2]\dot{q}^- \end{cases} \quad (7)$$

Eq. (7) is an expression for $\dot{q}^+$ in terms of $\dot{q}^-$, which should then be used to re-initialize the model (1). The impact model must account for the re-labeling of the robot coordinates (i.e. the swing leg becomes the new stance leg and vice versa), this can be expressed by

$$\begin{pmatrix} q^+ \\ \dot{q}^+ \end{pmatrix} = R(q)\begin{pmatrix} q^- \\ \dot{q}^- \end{pmatrix} \quad (8)$$

To summarize, the global impact model that includes both the jumps in velocities and the permutation of coordinates and velocities shortly writes

$$\begin{pmatrix} q^+ \\ \dot{q}^+ \end{pmatrix} = \Delta(q)\begin{pmatrix} q^- \\ \dot{q}^- \end{pmatrix} \quad (9)$$

where

$$\Delta(q) = \begin{pmatrix} R(q) & 0 \\ 0 & R(q)D(q) \end{pmatrix} \quad (10)$$

in which *R(q)* is the relabeling matrix with appropriate dimension.

5. Receding Horizon Control Scheme

In order to apply the receding horizon control scheme, first of all it is necessary to decompose all the degrees of freedoms of the model. This procedure is detailed in [13]. In this paper, the three independent degrees of freedoms of the model during swing phase can be subdivided into two parts as follows

$$z_1 = q_3 \in \Re; \quad z_2 = [q_1 \quad q_2]^T \in \Re^2 \tag{11}$$

Where z_1 and z_2 are indirectly and directly controllable variables, respectively. The sequence of impact instants with ground is denoted by $(t_k)_{k\in N}$ with $t_k = kt_f$ where t_f is the step duration.

Some target configuration $z_2^f \in \Re^2$ is chosen that is to be reached just before the impact instants t_k that is $z_2(t_k^-) = z_2^f$. This choice is fixed in all the forthcoming developments, in a way, z_2^f has to be considered as a design parameter which should be parameterized. In what follows, the following notations are used

$$\begin{aligned} &\Im_2 = \begin{pmatrix} z_2 \\ \dot{z}_2 \end{pmatrix} \in \Re^4; \quad \Im_2^f = \begin{pmatrix} z_2^f \\ \dot{z}_2^f\,(z_2^f, v_{Swing}) \end{pmatrix} \in \Re^4 \\ &\Im_1 = \begin{pmatrix} z_1 \\ \dot{z}_1 \end{pmatrix} \in \Re^2 \end{aligned} \tag{12}$$

Now, during the step, let us denote by $\eta > 0$ the remaining time before impact. One has the following dynamic for η

$$\dot{\eta} = -1 + \delta(\eta)t_f \tag{13}$$

in which $\delta(.)$ is the generalized impulse function. Consider a control sampling period $\tau_c > 0$ such that $t_f / \tau_c = N_c \in N$ (N_c is control horizon). We assume that the decision instants are on the interval of $[t_k, t_{k+1}]$ as

$$\tau_k^i = t_k + i\tau_c; \quad i \in \{0,1,..., N_c - 1\}; \quad k \in N$$

During the step, at each decision instant τ_k^i, a set of p-parameterized reference trajectories defined as follow

$$\Im_2^{ref}(\tau', \Im_2(\tau_k^i), \Im_2^f, \eta(\tau_k^i), p); \quad \tau' \in [\tau_k^i, t_{k+1}] \tag{14}$$

These trajectories are satisfied for all parameter value $p \in P$ the following initial and final conditions

$$\Im_2^{ref}(\tau_k^i, \Im_2(\tau_k^i), \Im_2^f, \eta(\tau_k^i), p) = \Im_2(\tau_k^i)$$
$$\Im_2^{ref}(\tau_{k+1}^i, \Im_2(\tau_k^i), \Im_2^f, \eta(\tau_k^i), p) = \Im_2^f \quad (15)$$

Namely, the reference trajectory $\Im_2^{ref}$ is updated at each decision instant τ_k^i to start at the present $\Im_2(\tau_k^i)$, and to join the desired final value $\Im_2^f$ just before next impact. It is worth noting that $p \in P$ is the remaining free parameter, once the constraints (15) have been structurally imposed, on some initial parameterization. The role of p is clearly to optimize the behavior of the indirectly controlled sub-state $\Im_1$. Indeed, imagine that a perfect tracking of the reference trajectory $\Im_2^{ref}$ is performed over $[\tau_k^i, t_{k+1}]$. What are the consequences of such tracking on the value of both $\Im_1$ and $\Im_2$ just before the *(k + 1)* impact?

For $\Im_2$, one would clearly have, because of the perfect tracking $\Im_2^{ref}(\tau_{k+1}^-) = \Im_2^f$. For the $\Im_1$ dynamic, the torso equation extracted from the dynamic model (1), should be considered

$$(\frac{1}{4}M_T l^2 + I_T)\ddot{q}_3 = \frac{1}{2}ml\cos(q_3)(\ddot{y} + g) - u_1 - u_2 \quad (16)$$

where M_T is the mass of the torso, *l* its center of mass length, u_1 and u_2 are the torques of the femurs. This dynamic can be written as

$$\dot{\Im}_1 = f(\Im_1, \Im_2^f, \Im_2(\tau_k^i), p) \quad (17)$$

Integrating (17) starting from the initial condition $(\tau_k^i, \Im_1(\tau_k^i))$ gives the predicted value of $\Im_1(\tau_{k+1}^-)$ just before next impact. This can be rewritten formally as follows

$$\hat{\Im}_1(t_{k+1}^- \mid \tau_k^i) = \psi(\Im_1(\tau_k^i), \Im_2(\tau_k^i), \Im_2^f, \eta(\tau_k^i), p) \quad (18)$$

Using Eq. (9) together with the predicted values $\Im_2^{ref}(\tau_{k+1}^-) = \Im_2^f$ and (18), an expression of the predicted value of $\Im_1$ just after impact can be derived as

$$\hat{\Im}_1(t_{k+1}^+ \mid \tau_k^i) = \psi^+(\Im_1(\tau_k^i), \Im_2(\tau_k^i), \Im_2^f, \eta(\tau_k^i), p) \quad (19)$$

The value of the reference trajectory's parameter $\hat{p}(\tau_k^i)$ is then given by the optimal solution of the following quadratic optimization problem

$$\hat{p}(\tau_k^i) = \min_{p \in P} \left\| \hat{\Im}_1(t_{k+1}^+ \mid \tau_k^i) - \Im_1^f \right\|_Q^2 \tag{20}$$

In each decision instant, the above optimization problem together with proper constraints is solved. Therefore, with keeping in hand the optimized value of p, the value of $\Im_1^f$ and $\Im_2^f$ is calculated and consequently the limit cycle of the model is defined.

6. Results and Discussions

The aim of simulation scenario presented here is the application of the receding horizon control described in previous section to biped model. Consider the three link model (Figure 1) with the following values of the parameters:

$$m = 5 \quad M_T = 10 \quad M_H = 15 \quad r = 1 \quad l = 0.5$$

corresponding to the mass of the legs, the mass of the torso, the mass of the hips, the length of the legs and the distance between the center of mass of the hips and the center of mass of the torso. The units are kilograms and meters.

Because presenting of several simulation results makes the article lengthy, we prefer to present some of the general results here. Figure 2 depicts the phase portrait of the un-actuated coordinate (torso), where we note the convergence to a limit cycle.

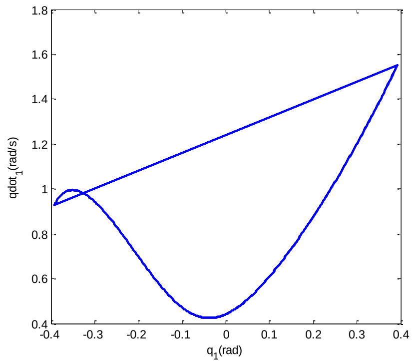

Figure 2. The phase portrait of the unactuated coordinate

Figure 3 displays the trajectory of angles of joints and angular velocity of them versus time. The applied torqueses over a few walking cycles are shown in Figure 4.

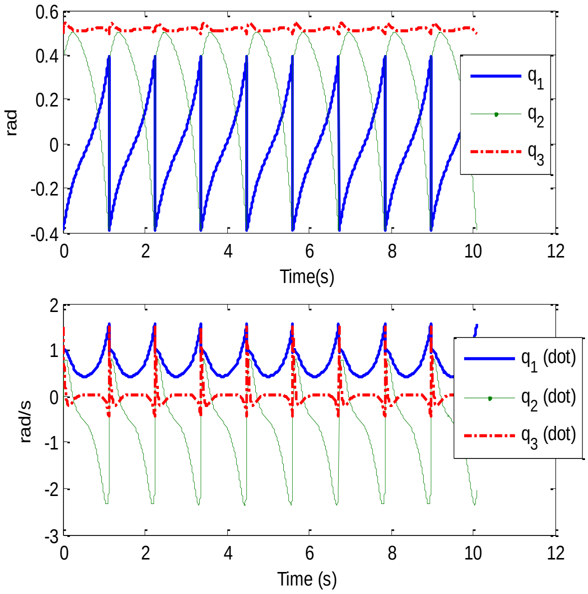

Figure 3. Angular trajectory and velocity of joints

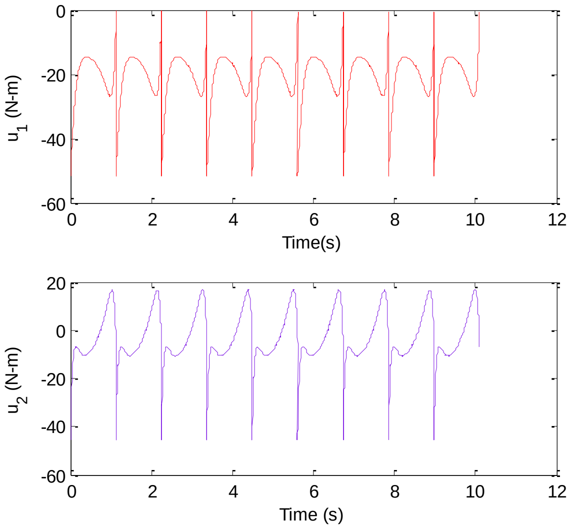

Figure 4. Applied torques

The simulation results clearly show the convergence of the all trajectories of the torso to an orbit which indicates the effectiveness of the method.

7. Conclusions

In this paper, for a three link planar limit cycle walker, a receding horizon based controller has been implemented. The controller is implemented to control the whole system to sustain its stable cyclic walking. The simulation results clearly showed the convergence of the all trajectories of the model to an orbital cycle. The approach inherently proves the stability of the motion and there is no need to use of Poincare-like argumentation in the proof of limit cycles. In the future work we plan to extend this method for disturbance rejection of the model under eternal disturbances.

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